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SOME CLASS ROOM EXPERIENCES IN TEACHING GEOMETRY.

By G. I. Hopkins.

As a sort of prelude I beg leave to offer a few thoughts suggested by an address to this association at the previous meeting wherein the utilitarian, or practical phase of the study of geometry was not only emphasized, but stoutly advocated, to the exclusion of the demonstrative.

This word "practical," it seems to me, has been more exploited and overworked than any other in the discussion of educational matters in recent years. Its advocates seem to convey the idea that no part of the education of a child is "practical" that cannot be put to immediate financial use. That if a pupil acquires a piece of information today at school, he should be able to market it to-morrow, and thus prove its utility. This word "utility" has become a sort of slogan by those, mostly specialists, who would limit the activities of the school to those things which the individual may at once commercialize.

Governor Hughes, at the Peary testimonial, said, "When the time comes that knowledge will not be sought for its own sake, and without hope of gain, then, indeed, will the race enter upon its decadence." But may not a course of study be useful even though it does not contribute to the weekly pay envelope? I think I agree with Mr. T. J. McCormack who has met this question, so far as mathematics is concerned, without dodging, and in a very frank manner. He says: "Not one tenth of the graduates of our high schools ever enter professions in which their algebra and geometry are applied to concrete realities; not one day in 365 is a high school graduate called upon to apply, as it is called, an algebraic or geometrical proposition,—Why, then, do we teach these subjects, if this alone is the sense of the word 'practical'? To me the solution of this paradox consists in boldly confronting the dilemma, and saying that our conception of the practical utility of these subjects must be readjusted, and that we have frankly to face the truth that the 'practical' ends we seek are, in a sense, ideal practical ends; yet such as have, after all, an eminently utilitarian value all their own, in the intellectual sphere."

At the same time I do not wish to be understood as attempting or desiring to belittle the importance of the practical applications of the ascertained truths of geometry. But the number of such truths susceptible of this application is surprisingly small, and the pupil must either accept them on faith or demonstrate them for himself. If the former, then he is not studying geometry, but simply doing some computing. If the latter, then he must begin at the beginning and take some logical sequence of theorems and prove them one by one until he reaches those he wishes to use. Anything less than this would result in a pseudo demonstration, partly mechanical and partly guess work.

The opponents of demonstrative geometry have declared, with considerable emphasis, that a boy who has taken such a course will be unable to think more logically when confronted with the problems of daily life, but I think he can and will. I heard one of them say not long ago that the study of geometry would not enable a man as a judge or juryman to sift and weigh evidence any better. In reply to this, a lawyer friend of mine, now a member of the supreme court of the state, said to me that he made it a practice to read Euclid's "Elements" through, at least, once a year, for the discipline it gave him in logical reasoning. Dr. Eugene Smith says: "Demonstrative geometry offers the best developed application of logic that we have, or are likely to have, in the school course; and to the pupil it offers a concrete example of close and exact reasoning that is not afforded by any of his other studies,—and it is extremely probable that the man who has had some drill in syllogisms, and who has learned to select the essentials in reaching his conclusions, has acquired habits in his reasoning that will help him in every line of work."

In the Perry Report Dr. Shaw said: "Let me pass on to say a word or two about the teaching of geometry as an academic training for general professional life. It has immense capabilities in that respect. If you consider how much of the effectiveness of an administrator depends upon the capacity for coördinating appropriately a number of different ideas, precise accuracy of definition, rigidity of proof, and sustained reasoning, strict in every step, and when you consider what substitutes for these things nine men out of ten without special training have to put up with, it is clear that the man with the geometrical training has incalculable advantages." At a teachers' convention more than thirty years ago when this question of utility was being mooted, I heard an enthusiastic teacher of the classics exclaim, that he believed that a man who had to dig potatoes would accomplish that task better for having studied Latin.

That, we will agree, is an extreme case; but if it be a fact that he works more contentedly, and while performing his manual labor his mind becomes reminiscent, and conjures up some of the beautiful passages of Virgil, of Cicero, and of Horace, and thus beguiles the weary hour, and is afforded a relief from the monotonous tedium of his uninteresting task, his time and energy spent upon the Latin classics certainly have a real value to him, though only an æsthetic value.

This leads directly to another phase of the subject, viz.: the pleasure one experiences in any line of study, and the satisfaction that comes from success in achievement. There is a peculiar joy, that often develops into a feeling of actual triumph, from the successful demonstration of a geometrical theorem that has been somewhat puzzling and baffling. And I know that I am not exaggerating when I say, that a considerable majority of the pupils in demonstrative geometry sometimes experience that exultant triumph, if the subject is taught in the right way, and the pupils are not robbed of their opportunity. To quote from Dr. Smith again, "To deny that our pupils derive this pleasure from the study of geometry is to confess ourselves poor teachers. It is surely partly æsthetic, the pleasure that comes from the symmetry of form, and the delight of fitting things together. As the pupil advances in the study of geometry this pleasure is augmented, and probably lies chiefly in the mental uplift that it brings, as he recognizes he is in contact with absolute truth, and the approach he is making to the Infinite."

There is just one more thought that is suggested in this connection and which is here presented without elaboration. The study of geometry affords one of the most effective means for correcting habits of carelessness, inaccuracy, forgetfulness, slovenliness, and inattention, and this is by no means the least in importance.

But I do not need to argue this question further to the members of this association. The only phase of the subject on which we might differ would be the method of presentation. And here my class room experiences covering a period of thirty years must be my argument. The prevailing methods during that time can be classified as follows:

1. The memorizing of proofs with no original work, and little or no instrumental practice.
2. Memorizing the principal proofs, with exercises for original work both demonstrative and instrumental.
3. The syllabus method requiring all original work, or practically all.
4. That which may be termed the mechanical, which makes it merely a tool for the mechanic and artisan.

My first attempt was with a class in the second year in the high school, they having had elementary algebra the previous year. We used Wentworth's text book which had been copyrighted a few years previous. I became deeply interested in the work at once because of the exercises for original work, which was a new departure in geometry text, and although not an ideal one, it was so much superior to the prevailing texts that it deserved the immense success which it attained. The members of the class soon became imbued with the same enthusiasm which I felt over the exercises, especially so as they learned that many of those exercises were more difficult than the proofs printed for them to learn. When we had reached the subject of quadrilaterals I resolved to try an experiment. I had already satisfied myself that a large majority of the class could prove those theorems unaided that were printed for them to learn, I was also satisfied that the remainder would profit more by the attempt, even though but partially successful, than to memorize the written proof. Acting upon that conviction I collected the books from the pupils and arranged a syllabus for them, following closely, of course, Wentworth's text. The success attained was beyond my fondest hopes, and since then we have used a syllabus of some kind. When one stops to think of it, why should the pupil be asked to read over another's proof of a theorem to memorize and then be asked to prove unaided a more difficult one?

Consistency, it seems to me, requires that he should, at least, have the opportunity for a trial at the former. I cannot see as there is a single argument in favor of the memorizing plan. I know it is expected that the instructor will try to devise various schemes to prevent the pupil from memorizing, like making a change in the form of the figure, or tipping it over, or changing the letters. When the pupil discovers this device to thwart him, he in turn prepares his defence, and so the time and energies of both pupil and instructor are wasted in a fruitless intellectual contest to see which shall outwit the other. How much more sensible and profitable must it be to give the pupil his opportunity for investigation unhampered. As well might the teacher of Virgil, for instance, place a metrical translation in the hands of his pupils to read, as for the teacher in geometry to discredit the pupil's ability by furnishing him a printed proof.

The results of using the syllabus method have been most satisfactory and beneficial to both pupil and instructor. To the pupil mainly for the increased power and independence acquired in all mathematical investigations, and for the pleasure and satisfaction that comes from successful achievement.

The interest developed in certain proofs has been somewhat remarkable. For example: as soon as we had finished that series of theorems relating to the measurement of angles, the pupils were given the following one.

"The angle formed by two tangents to the same circle is double the angle formed by the diameter drawn from one point of contact and the chord joining both points of contact."

The result was eight or ten different proofs, arising mostly from variations in construction. Subsequent classes have also had it assigned to them, and so far I have received more than *sixty* different demonstrations. Just suppose the author of a text book had provided a printed proof of that theorem for the pupil to learn. The Pythagorean theorem was also of much interest, and I have been presented with, at least, three new proofs of that. When I entered college, no geometry was required for preparation, so that my course in plane geometry was in the freshman year, with Davies' "Legendre" the text book. Only once did our professor offer us anything outside the text. Only two members of the class succeeded in finding a proof, one of them myself, and it took all the spare time we could give to it for two months before we succeeded. Three or four years after I began to use the syllabus I gave that same one to the class and within a week eight members handed me a correct proof. I have always consoled myself with the belief that it was not because they were so much better scholars than I was, but because of the difference in the method of presentation.

Besides this independence of text book and development of power to think logically, there is fostered an attitude of keen criticism, both of phraseology and statement, so that slip-shod, illogical proofs are quickly detected. Some of them have been much quicker than myself to detect inconsistencies both in definitions and phraseology of theorems. One of the first to fall under the ban of criticism was the very elementary one, as it now appears in most of the recent text books, viz.: "The sum of all the angles about a point on one side of a straight line passing through that point equals two right angles." A boy went to the board, drew the diagram, having four lines drawn from a point in the line. In stating what was to be proven, he made such a long list of angles that he was interrupted before he had completed the list by objections from his classmates. Of course, the theorem means simply the sum of the five angles, but he had taken it literally, as he had a right to do. Had he been allowed to continue without objection he would doubtless have indicated the sum of the entire fourteen angles. The discussion resulted speedily in correcting that phraseology to conform to the intended meaning by inserting the word "consecutive" before the word "angles." A similar correction was made in the theorem relating to the sum of the angles about a point. They had got a pretty clear idea of what the term "adjacent angles" meant after a few illustrations, so it was not very surprising to find them critizing the phraseology of the theorem, "If a side and two adjacent angles of one triangle are equal to a side and two adjacent angles of another, etc." It was a unanimous decision that there were no "adjacent angles" in a triangle; and they concluded that "two angles and included side," was entirely satisfactory and adequate.

About twenty years ago I was developing the subject of loci by the use of a moving point. During a pause a hand was raised, and the owner permitted to inquire. He said he didn't see how a point could move. That led to a general discussion, and he based his objection on the definition of a point; viz., "A point is that which has position, but not extension." He claimed that if a point moved it parted with the only property which it possessed, *i. e.,* position. We had to agree that motion could be predicated only of material things, but that the idea was useful, and that, although not strictly in accord with fact, mathematicians felt compelled to utilize it. That young man is now assistant professor of mathematics in one of our New England colleges.

When the term "straight angle" was first proposed, I thought if others were going to use it we might as well, although I did not favor it, and saw no real need of it. However, its use was allowed but not encouraged, and the consequence was, little was heard of it in the class. About ten years ago, however, the matter was brought to our notice in quite an emphatic manner. I had asked a lad what a triangle was, and he replied very promptly, "A triangle is a plane figure having three sides and six angles." He was asked to go to the board, draw his figure and explain. He readily complied; and his explanation was, "It has the three sides *AB, BC* and *AC,* the three oblique angles *A, B* and *C,* and the three straight angles *AB, BC* and *AC.*" I have always suspected that he took that way to show how ridiculous the term "straight angle" was from his point of view, after the several illustrations of an angle. At any rate I came to a decision very quickly, and directed them to use their pencils and obliterate the paragraph relating to the straight angle.

There is another difficulty which some of the recent text books present that use that term, when the theorems are reached relating to exterior angles, the latter not having been defined. There has been no reference to straight angles in my classes since then, and since we have the right angle, degree with its subdivisions, and radian, there is no need of another unit of angle measurement. About a year ago I sent letters of inquiry to about fifty professors of mathematics, both pure and applied, in New England and middle states, asking their opinion about the use of that additional unit, and without exception they declared their disapproval of its use, and one emphasized his objection by calling it "mathematical slang"! When the student takes up trigonometry he will, of course, be confronted with angles of 180°. But that is a very different thing from showing a pupil in his early 'teens a straight line and then in the next paragraph calling that same line an angle. That is as reprehensible as to state to the same young beginner that a straight line is a variety of curve, as occurs in a recent text-book. Words and phrases should convey some distinct and permanent meaning to the young beginner, and all inconsistent and perplexing phrases banished.

"Sect" and "ray" are terms that have also been introduced, but they serve only to confuse the beginner, however serviceable the expert mathematician may find them; and I agree with Dr. Smith when he says that the use of these terms is "pedantic." Those texts which make use of these terms in the plane geometry must see that there is the same necessity for a term to designate a terminated plane in the solid geometry; but they have not provided it, and so are open to the charge of inconsistency. In the words of a prominent member of our association, "Why introduce unnecessary elements of confusion when there are plenty of difficulties with the clearest notation?" My experience is that pupils have very little difficulty with the term "line," whether terminated or indefinite is meant.

Pupils, as a rule, do not like the indirect method of proof. Because of this I have offered special inducements to pupils to discover direct proofs of those theorems of which we knew only the indirect. The result is three or four such direct proofs. I doubt very much if this could have been accomplished without the syllabus plan.

Pupils also dislike many corollaries, and, I must confess, I agree with them. We must not forget that what may rightly be an easy corollary to the mature and expert mathematician, is by no means such to the young beginner in his early 'teens groping his way. And it is for these that the text book should be written. One recent text has carried this matter of corollaries to an absurd extent by giving *thirteen* corollaries to the theorem about the angle sum of a triangle. In view of all this evidence, which is the true, scientific, pedagogical, method of presenting the subject? Instead of printing the proofs why not allow the pupil to utilize his time and energies in the endeavor to originate them himself? This question is particularly pertinent when we consider the number of essential theorems compared with the number of exercises that are exercises only. The first six books of Euclid, excluding the fifth which treats of proportion, contains 148. Dr. Eugene Smith thinks a syllabus of 140 will suffice. The Harvard syllabus contains 102, but without any problems in construction. The syllabus of our association contains 110, with the suggestion that it could profitably be increased. Why not decide upon a syllabus of about 150 or 160, and let the pupil try his skill and mettle on those, rather than print these for him to learn and then require of him the proofs of 300 or 400 exercises that he can never make any use of?

It is difficult for the teacher, to whom the subject is familiar, to appreciate the standpoint of the beginner, especially in mathematics, unless he is constantly on the alert. The tendency is to get impatient over the seemingly petty difficulties which they encounter. But we must remember that those things which have ceased to be difficulties to the mature and experienced mind, through long familiarity, may be, and very often are, real and present difficulties to the beginner.

We should not get discouraged over the slowness with which they get under way, for their reasoning faculties develop slowly, and they must be allowed time to ponder. Let us not forget that the chief aim in the study of mathematics should be "to strengthen and develop the pupil's faculties, and to supply a method of reasoning applicable to other subjects and to other conditions, rather than to furnish him with an instrument for solving problems, believing that the shortsightedness of a narrow education, of an education that teaches only machines to a prospective mechanic, and agriculture to a prospective farmer, and cooking and dress-making to the girl, and that would exclude all mathematics that is not utilitarian in the narrow sense, cannot endure."

And above all else in importance, let us not rob the pupil of his opportunity by placing before him the printed proof until he has, at least, had a chance, to struggle with it. Even then, a suggestion or two, or a well directed question, will afford him all the assistance he needs, in a large majority of cases. No permanent benefit can come to the pupil's mind when all he has to do is to yield ready assent to the statements supplied by another. Mental power is acquired only by a mental struggle, and this is impossible for the beginner in geometry when the proofs are furnished him, since the opportunity for the requisite mental struggle is denied him. Consistency demands that many of the so-called exercises be eliminated, and the proofs of those in the regular sequence be suppressed. In no other way can the pupil be given his rightful opportunity for both the pleasure and profit that comes from a rational study of geometry.

Since the above address was given there has come to my notice in the MATHEMATICS TEACHER an announcement of a book entitled "Inductive versus Deductive Methods of Teaching," by W. H. Winch. In this announcement occurs the following statement: "In five different schools of London, attended by children of different social classes, a series of experiments were made to test the relative values of inductive and deductive methods of teaching as applied to geometrical definitions.

"It was found that in three of the schools tested those taught deductively could reproduce precisely what they had been taught better than those taught inductively. In the other two the results were about even. In testing to find out which of the two methods gave the better results when the children were tested on new material it was shown that in all five schools those taught inductively did better work. This would seem to be a strong argument in favor of the syllabus method of teaching geometry."

HIGH SCHOOL,
MANCHESTER, N. H.